

# Multi-fidelity aerodynamic shape optimization of suspension bridge deck with railings and vortex mitigation devices

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## SUMMARY:

This paper presents an application of the multi-fidelity shape optimization method to the aerodynamic design of a single box girder for suspension bridges. Since Computational Fluid Dynamics (CFD) is a computationally intensive process, high-fidelity physics model simulation at lower computational cost is desirable, especially during the initial design phase. The variable fidelity surrogate modeling method is adapted to alleviate this high computational cost with the use of a large number of low-fidelity CFD models. The high-fidelity CFD models include detailed geometry of railings and vortex mitigating devices, which affect the aerodynamic performance of the bridge significantly by altering the flow field around the deck. Once the force coefficients are defined in the design domain, a shape-optimization problem is formulated under flutter constraint based on the quasi-steady theory. The method is applied to a proposed suspension bridge concept and is proven to be computationally efficient compared to an approach using solely high-fidelity function evaluations.

*Keywords: optimization, multi-fidelity, flutter*

## 1. AERODYNAMIC MULTI-FIDELITY FORMULATION

The assessment of aerodynamic response of suspension bridges is essential for the design because of their vulnerability to wind-induced vibrations. Since the deck shape plays a key role in aerodynamic behavior, the knowledge of aerodynamic response according to its shape in the initial design phase will be of great value. Aerodynamic force coefficients can be obtained either experimentally or computationally. Nonetheless, performing numerous expensive experimental tests to cover potential design domain is not realistic. Force coefficients can be obtained by Computational Fluid Dynamic (CFD) although the computational cost of precise CFD simulations is very high. Multi-fidelity co-Kriging makes use of a greater quantity of readily available low fidelity CFD data corrected by a small amount of expensive data. Thus, this method permits the saving of computational cost without compromising accuracy. In this study, a series of CFD simulations is performed using precise HF models and approximate LF models. Co-Kriging models of aerodynamic coefficients are constructed in the entire design domain, which makes it

possible to carry out design optimization of the deck shape. Firstly, the model convergence using different number of HF models are studied; then co-Kriging models are compared to the traditional Kriging models. Unlike other studies of deck shape optimizations, the deck details such as guide rails and vortex mitigating devices, are considered for the CFD simulations. The CFD results of the original design was validated by experimental tests. Finally, design optimization of the deck shape was carried out considering flutter constraint.

The Kriging method developed by Krig in 1951 and improved by Sacks et al. in 1989 provides predicted values at a point in the design domain based on a set of observed response at sampled points. Co-Kriging is an extension of the Kriging method. When a greater quantity of quick and simple estimations of the expensive function are available, they can be coupled with a small quantity of the expensive data to enhance the accuracy of the surrogate model.

By defining a low-fidelity and high-fidelity data set as  $[\mathbf{X}_L, \mathbf{Y}_L]$  and  $[\mathbf{X}_H, \mathbf{Y}_H]$  and following the auto-regressive model of Kennedy and O'Hagan (2000), we can approximate the high-fidelity function as:

$$Z_H = \rho Z_L(\mathbf{x}) + Z_D(\mathbf{x}) \quad (1)$$

where  $Z_L$  and  $Z_H$  represents Gaussian process of local features of the low-fidelity and high-fidelity data,  $\rho$  is a scaling factor and  $Z_D$  is a Gaussian process that represents the difference between  $Z_H$  and  $\rho Z_L$ .

The covariance can be expressed as (Forrester et al., 2007):

$$\mathbf{C} = \begin{bmatrix} \sigma_L^2 \Psi_L(\mathbf{X}_L, \mathbf{X}_L) & \rho \sigma_L^2 \Psi_L(\mathbf{X}_L, \mathbf{X}_H) \\ \rho \sigma_L^2 \Psi_L(\mathbf{X}_H, \mathbf{X}_L) & \rho^2 \sigma_L^2 \Psi_L(\mathbf{X}_H, \mathbf{X}_H) + \sigma_D^2 \Psi_D(\mathbf{X}_H, \mathbf{X}_H) \end{bmatrix} \quad (2)$$

where  $\Psi_L$  and  $\Psi_D$  denote matrix correlation of the low fidelity and the difference data,  $\sigma_L^2$  and  $\sigma_D^2$  are variances of  $Z_L$  and  $Z_D$ . In the case of co-Kriging, we have two correlations: one is for the LF data,  $\Psi_L$  and the other for the difference function,  $\Psi_D$ . The hyper parameters of LF data can be obtained by maximizing the concentrated natural log likelihood of LF data using the standard Kriging. The difference function is defined as:

$$\mathbf{d} = \mathbf{y}_H - \rho \mathbf{y}_L(\mathbf{X}_H) \quad (3)$$

Then the co-Kriging prediction of the high-fidelity function is expressed as:

$$\hat{y}_H(x^{n+1}) = \hat{\mu} + \mathbf{c}^T \mathbf{C}^{-1} (\mathbf{y} - \mathbf{1} \hat{\mu}) \quad (4)$$

where  $\mathbf{c} = \begin{pmatrix} \hat{\rho} \hat{\sigma}_L^2 \Psi_L(\mathbf{X}_L, \mathbf{x}^{(n+1)}) \\ \hat{\rho}^2 \hat{\sigma}_L^2 \Psi_L(\mathbf{X}_H, \mathbf{x}^{(n+1)}) + \hat{\sigma}_L^2 \Psi_D(\mathbf{X}_L, \mathbf{x}^{(n+1)}) \end{pmatrix}$  and  $\hat{\mu} = \mathbf{1}^T \mathbf{C}^{-1} \mathbf{y} / \mathbf{1}^T \mathbf{C}^{-1} \mathbf{1}$

$\Psi_L(\mathbf{X}_H, \mathbf{x}^{n+1})$  denotes the correlations of  $\Psi_L$  between  $\mathbf{X}_H$  and the new point  $\mathbf{x}^{n+1}$ .

## 2. MULTI-FIDELITY MODELS FOR THE BRIDGE OPTIMIZAITON CONSIDERING FLUTTER

In this work, the flutter wind speed for the considered bridge deck geometries is determined by a multi-modal response analysis. The aeroelastic forces acting on the bridge deck are modelled based on static force coefficients obtained by CFD simulations for different angles of attack. This provides a fully numerical approach to flutter computation. The time-average, static load coefficients are considered to adequately represent the wind-structure interaction for streamlined deck sections at high reduced wind velocities as discussed in Wu and Kareem (2013).

The initial deck geometry considered for the optimization problem is shown in Figure. 1. The numerical CFD model contains the deck details of four sets of railings, guide vanes, and spoiler. The variation of the deck geometry was obtained by moving the leading and the trailing edges symmetrically both horizontally to change the deck width and vertically to change the angles  $\theta_1$  and  $\theta_2$  shown in the figure. The deck height was maintained constant. A total of 40 designs of experiment was chosen for the LF models using Latin Hypercube Sampling (LHS) while 17 HF models were chosen from the LF design using the exchange algorithm (Cook R.D. 1980).

For creating multi-fidelity level models, one way is to use different mesh-resolution models. Another method may be using partially converged CFD simulations, which correlate well with the converged counterparts (Forrester et al. 2006). These two options were considered in this research and the results were compared.

CFD simulations were performed using OpenFoam v.6.0 with 2D URANS approach with  $k-\omega$  Shear-Stress Transport (SST) turbulence model. The HF models consist of approximately 300,000 cells with boundary layers attached around the deck while coarsely meshed LF models with approximately 20,000 cells use the wall function as near wall treatment.

The force coefficients obtained by the HF CFD simulations of the initial design were validated experimentally using a sectional model of 1/100 in the wind tunnel at the University of Coruña.

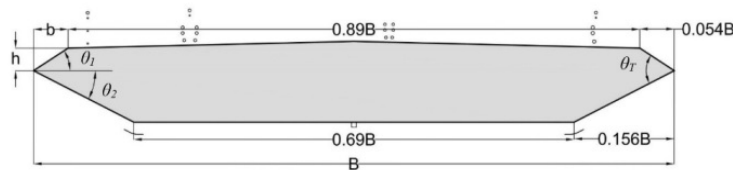


Figure 1. Initial design of the bridge deck section

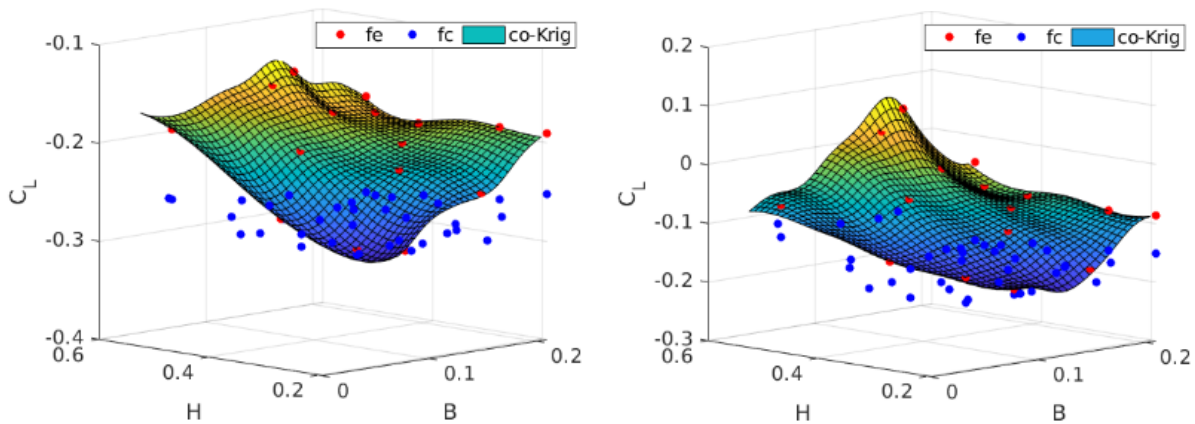


Figure 2.  $C_L$  Co-Kriging model with 17 HF points at a)  $0^\circ$  and b)  $2^\circ$  angle of attack;  $H$  and  $B$  at model scale (1:100)

A convergence study of multi-fidelity models was performed using four different sets of HF data points and three independent validation points. Figure 2 shows the co-Kriging surface of lift coefficient at 0 and 2 degrees of angle of attack (AoA) with 17 HF points.

Once the co-Kriging surfaces are obtained for  $C_L$ ,  $C_D$  and  $C_M$  in the design domain, a shape optimization of the bridge deck can be performed. The objective function was to maximize the flutter speed, which was computed based on the force coefficients estimated from the co-Kriging surrogate models and quasi-steady formulation. The frequencies and mode shapes of the bridge were obtained from an Abaqus finite element model. Additionally, the design constraints of maximum main cable stress and the maximum vertical displacement of the bridge deck under traffic overload cases were considered (Kusano et al. 2020).

The preliminary design of the bridge for crossing the Julsundet Fjord in Norway was used as a study case. It is a suspension bridge with a main span of 1.6 km. The deck section is 32 m wide and 4 m high aerodynamic box girder. The baseline geometry used for the CFD simulations followed that of a 1:50 scale model. Further description of the bridge can be found in a report from the Norwegian Public Road Administration (2015).

### 3. CONCLUSIONS

Multi-fidelity co-Kriging models were developed for the aerodynamic shape optimization of suspension bridge decks. While maintaining accuracy, this method alleviates the high computational cost of high-fidelity CFD models by using readily available low-fidelity models. For the estimation of force coefficients accurately, it is important to include detailed deck geometry of railings and aerodynamic appendices for the HF models. The method was successfully applied to the shape optimization problem of a suspension bridge deck in Norway to demonstrate the efficiency and feasibility of the method.

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